

Meta-Stable Brane Configurations with Seven NS5-Branes

Changhyun Ahn

Department of Physics, Kyungpook National University, Taegu 702-701, Korea

ahn@knu.ac.kr

Abstract

We present the intersecting brane configurations consisting of NS-branes, D4-branes (and anti D4-branes) and O6-plane, of type IIA string theory corresponding to the meta-stable non-supersymmetric vacua in four dimensional $\mathcal{N} = 1$ supersymmetric $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory with a symmetric tensor field, a conjugate symmetric tensor field and bifundamental fields. We also describe the intersecting brane configurations of type IIA string theory corresponding to the non-supersymmetric meta-stable vacua in the above gauge theory with an antisymmetric tensor field, a conjugate symmetric tensor field, eight fundamental flavors and bifundamentals. These brane configurations consist of NS-branes, D4-branes (and anti D4-branes), D6-branes and O6-planes.

1 Introduction

The dynamical supersymmetry breaking in meta-stable vacua [1] arises in the $\mathcal{N} = 1$ SQCD with massive fundamental flavors where the masses for quarks are much smaller than the dynamical scale of gauge sector. See also the review paper [2] for recent developments on supersymmetry breaking. In the magnetic dual theory with massless quarks, the superpotential has a cubic interaction term between the meson and quarks. For the corresponding brane configuration where there exist two NS-branes, D4-branes and D6-branes, see the reference [3]. Due to the extra mass term for quarks in the magnetic superpotential with massive quarks, not all F-term equations can be satisfied. Then the supersymmetry is broken by rank condition [1]. Therefore, both the mass term for the quarks in the superpotential and the magnetic dual procedure are crucial in order to construct the new meta-stable supersymmetry breaking vacua. The meta-stable brane configurations of type IIA string theory corresponding to $\mathcal{N} = 1$ SQCD with massive fundamental flavors have been found in [4, 5, 6]. See also [7] which has dealt with the case where there exists an extra adjoint matter.

To construct brane configurations that we will describe, it is very useful to decompose $9 + 1$ dimensional space time as

$$(x^0, x^1, x^2, x^3), \quad (x^4, x^5), \quad x^6, \quad x^7, \quad (x^8, x^9).$$

Then $3 + 1$ dimensions labeled by (x^0, x^1, x^2, x^3) are common all the branes and we introduce two complex planes

$$v = x^4 + ix^5, \quad w = x^8 + ix^9.$$

There are two types of NS-branes which stretch in (x^0, x^1, x^2, x^3) as well as following directions

$$NS5 - \text{brane} \quad : \quad v,$$

$$NS5' - \text{brane} \quad : \quad w.$$

These branes preserve $\mathcal{N} = 2$ supersymmetry in $3 + 1$ dimensions. The NS-brane configuration of interest is given by a single NS5-brane extended v and localized at $x^6 = 0$ and two NS5'-branes at $(v = v_1, x^6 = y_1)$ and $(v = v_2, x^6 = y_2)$ respectively. Now we add N'_c D4-branes stretched between the NS5-brane and one of the NS5'-branes and N_c $\overline{D4}$ -branes stretched between the NS5-brane and the other NS5'-brane in order to break the supersymmetry. These D4-branes and $\overline{D4}$ -branes stretch in (x^0, x^1, x^2, x^3) and have finite interval along x^6 -direction. Then this brane construction is drawn in Figure 1A found in [8]. In the Figures of this paper, the vertical axis is v -direction, the horizontal axis is x^6 -direction and the other orthogonal direction coming out of the page is w -direction.

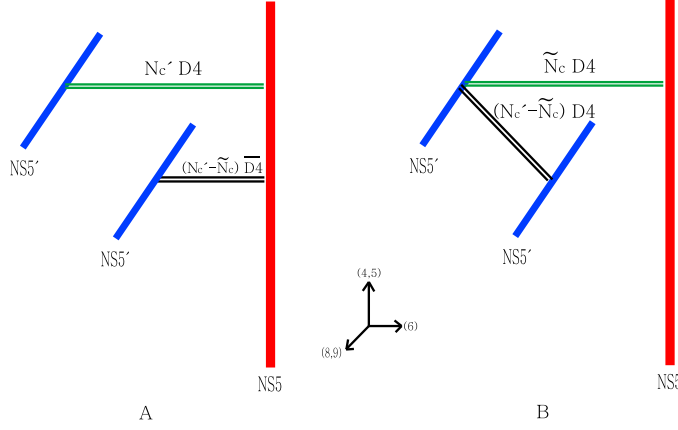


Figure 1: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(\tilde{N}_c = N'_c - N_c) \times SU(N'_c)$ with D4- and $\overline{D4}$ -branes(1A) and with a misalignment between D4-branes(1B) when the NS5'-branes of 1A are close to each other.

Recently, Giveon and Kutasov [8] have found the type IIA brane configuration consisting of two NS5'-branes, a single NS5-brane, D4-branes and anti-D4-branes($\overline{D4}$ -branes). The gravitational interaction for the D4-branes in the background of the NS5-branes has led to the phase structures in different regions of the parameter space. The meta-stable vacua of [1] occur in some region of parameter space when the D4-branes and $\overline{D4}$ -branes can decay(by taking the distance between the NS5'-branes very small) and the geometric misalignment of flavor D4-branes arises and this feature is shown in Figure 1B.

One advantage of this construction is the fact that there are no D6-branes and this enables us to write the magnetic superpotential in simple form. One disadvantage of this construction is that one has to increase the number of NS-branes, compared with the description of [4, 5, 6], since the role of D6-branes is replaced by the role of NS5'-brane. The mass term in the magnetic superpotential in this approach corresponds to the relative displacement of two NS5'-branes along the 45 directions and the dual quarks can be represented by the bifundamentals of product gauge group since there are three NS-branes. When the NS5'-brane is replaced by coincident D6-branes, then the magnetic meta-stable brane configuration will lead to the one in [4, 5, 6].

Adding an orientifold 4-plane only to the brane configuration of [8] implies that the gauge group is a product of a symplectic group and an orthogonal group and the geometric misalignment of flavor D4-branes [9] together with a replacement of NS5'-branes by coincident D6-branes leads to the brane configuration of [10]. In the standard supersymmetric electric brane configuration of type IIA string theory, the $\mathcal{N} = 1$ product gauge group theory [11, 12] is realized by three NS-branes, two kinds of D4-branes, and two kinds of D6-branes [13].

When an orientifold 6-plane is added into the brane configuration of [8], the gauge group is a product of two unitary groups with extra matters as well as bifundamentals. In this case, the type IIA brane configuration consists of five(the number of NS-branes plus the number of kinds of D6-branes in [13]) NS-branes, D4-branes, $\overline{D4}$ -branes and O6-plane. Similarly, the geometric misalignment of flavor D4-branes [9] with a replacement of NS5'-branes by coincident D6-branes leads to the brane configurations of [14] or [15] depending on the O6-plane charge.

One can generalize the work of [8] further by adding more NS-branes to the brane configuration of [8]. For the $\mathcal{N} = 1$ triple product group gauge theory, the supersymmetric electric brane configuration in type IIA string theory consists of four NS-branes, three kinds of D4-branes, and three kinds of D6-branes [13, 16] in the conventional brane configuration. Then the triple product gauge group theory with bifundamentals only can be realized by four NS-branes, three kinds of D4-branes and $\overline{D4}$ -branes [17] since there are no quarks. The meta-stable vacua of [18] occur in some region of parameter space when the D4-branes and $\overline{D4}$ -branes can decay and the geometric misalignment of flavor D4-branes arises.

Adding an orientifold 4-plane only to the brane configuration of [17] leads to the fact that the gauge group is a triple product of a symplectic group and an orthogonal group alternatively and the geometric misalignment of flavor D4-branes [17] together with a replacement of NS5'-branes by coincident D6-branes reduces to the brane configuration of [19]. One can add an orientifold 6-plane into the brane configuration of [13, 16] together with extra outer NS-branes in order to keep the number of gauge group factor. Then the type IIA brane configuration consists of six NS-branes, D4-branes, $\overline{D4}$ -branes and O6-plane. The geometric misalignment of flavor D4-branes [17] with a replacement of NS5'-branes by coincident D6-branes leads to the brane configurations of [18]. Different O6-plane charge will give rise to other brane configuration of [18] by a replacement of NS5'-branes by D6-branes.

In this paper, we present the intersecting brane configurations of type IIA string theory corresponding to the new meta-stable nonsupersymmetric vacua in four dimensional $\mathcal{N} = 1$ triple product gauge theory with matters. For the $\mathcal{N} = 1$ product gauge group theory with extra matters as well as bifundamentals, the supersymmetric electric brane configuration in type IIA string theory consists of five NS-branes, two kinds of D4-branes, and two kinds of D6-branes as well as O6-plane [20].

When the two outer NS5-branes are added into the brane configuration of [20] and removing the D6-branes completely, the gauge group is a product of three unitary groups with extra matters as well as bifundamentals. In this case, the type IIA brane configuration consists of seven NS-branes(i.e., the number of NS-branes plus the number of kinds of D6-branes in [20]),

D4-branes, $\overline{D4}$ -branes and O6-plane. For a given supersymmetric electric gauge theory which does not have any superpotential, one takes both the mass deformation for bifundamentals and its Seiberg dual. Then we construct the meta-stable brane configurations in type IIA string theory. Furthermore, the same gauge theory with more complicated matters is realized by seven NS-branes, D4-branes, $\overline{D4}$ -branes, D6-branes and two kinds of O6-planes.

In section 2, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory with a symmetric tensor field, a conjugate symmetric tensor field and bifundamental fields and deform this theory by adding the mass term for the bifundamental. We construct the three different dual magnetic theories by taking the Seiberg dual for each gauge group factor. Then we consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory.

In section 3, we discuss the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory with an antisymmetric tensor field, a conjugate symmetric tensor field, eight fundamental flavors and bifundamentals and deform this theory by adding the mass term for the bifundamental. Then we construct the corresponding dual magnetic theories by taking the Seiberg dual for each gauge group factor. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory.

In section 4, we summarize what we have found in this paper and make some comments for the future directions ¹.

2 Meta-stable brane configurations-I

The type IIA brane configuration corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group

$$SU(N_c) \times SU(N'_c) \times SU(N''_c) \quad (2.1)$$

and with a symmetric tensor field S charged under $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + 1), \mathbf{1}, \mathbf{1})$, a field F charged under $(\mathbf{N}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$, a field G charged under $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}}''_c)$, and their conjugates \tilde{S} , \tilde{F} and \tilde{G} can be described by the left $NS5'_L$ -brane(012389), the NS5-brane(012345), the right $NS5'_R$ -brane(012389)(and their mirrors), the middle NS5-brane(012345), N_c -, N'_c - and N''_c -color D4-branes(01236)(and their mirrors) as well as the $O6^+$ -plane(0123789). The $O6^+$ -plane acts as

¹A replacement of D6-branes with NS5'-brane corresponds to the gauging of the flavor group of the gauge theory realized on the D4-branes and this replacement might be useful to construct the phenomenological model building.

$(x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)$ and has RR charge $+4$ ². See also the relevant works [21, 22] which have dealt with the gauge theory realized by the first factor in (2.1) with a symmetric tensor, a conjugate symmetric tensor and fundamentals.

Let us place an $O6^+$ -plane at the origin $x^6 = 0$ and the x^6 coordinates for the $NS5'_L$ -brane, the NS5-brane and the $NS5'_R$ -brane are given by $x^6 = y_1, y_1 + y_2, y_1 + y_2 + y_3$ respectively. The locations for their mirrors can be understood similarly. The N_c D4-branes are suspended between the $NS5'_L$ -brane and its mirror, the N'_c D4-branes are suspending between the $NS5'_L$ -brane and the NS5-brane(and their mirrors), and the N''_c D4-branes are suspended between the NS5-brane and the $NS5'_R$ -brane(and their mirrors). The fields S and \tilde{S} correspond to 4-4 strings connecting the N_c -color D4-branes with $x^6 < 0$ with N_c -color D4-branes with $x^6 > 0$. The fields F and \tilde{F} correspond to 4-4 strings connecting the N_c -color D4-branes with N'_c -color D4-branes while the fields G and \tilde{G} correspond to 4-4 strings connecting the N'_c -color D4-branes with N''_c -color D4-branes. We draw this $\mathcal{N} = 1$ supersymmetric electric brane configuration in Figure 2A for the vanishing mass for the fields G and \tilde{G} .

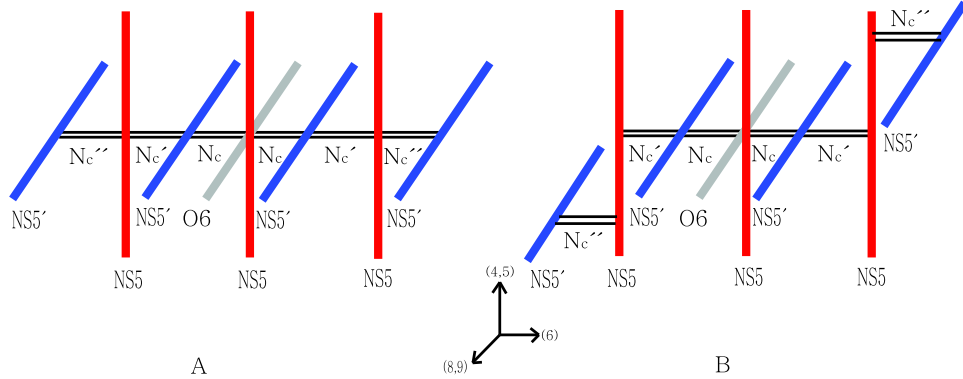


Figure 2: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ and a symmetric tensor S , a conjugate symmetric tensor \tilde{S} , the bifundamentals F, \tilde{F}, G and \tilde{G} with vanishing(2A) and nonvanishing(2B) mass for the fields G and \tilde{G} . The $NS5'_R$ -brane together with N''_c D4-branes is moving to $+v \equiv x^4 + ix^5$ direction in 2B(and their mirrors to $-v$ direction).

The gauge couplings of $SU(N_c)$, $SU(N'_c)$ and $SU(N''_c)$ are given by a string coupling constant g_s , a string scale ℓ_s and the x^6 coordinates y_i for three NS-branes through

$$g_1^2 = \frac{g_s \ell_s}{y_1}, \quad g_2^2 = \frac{g_s \ell_s}{y_2}, \quad g_3^2 = \frac{g_s \ell_s}{y_3}.$$

²When we say about NS-branes(i.e., NS5-brane or NS5'-brane) in this section, those NS-branes are located at the region of $x^6 > 0$. We assume that their mirrors behave according to this \mathbf{Z}_2 symmetry of $O6^+$ -plane. That is, there exist three NS-branes: $NS5'_L$ -brane, NS5-brane and $NS5'_R$ -brane from Figure 2A.

As y_3 goes to ∞ , the $SU(N_c'')$ gauge group becomes a global symmetry and the theory leads to the super QCD with the gauge group $SU(N_c) \times SU(N_c')$ and N_c'' flavors in the fundamental representation of second gauge group factor [20].

Now we describe three different magnetic dual theories by taking each corresponding mass deformation.

2.1 $\mathcal{N} = 1$ $SU(N_c) \times SU(\tilde{N}_c') \times SU(N_c'')$ magnetic theory

Let us take the mass deformation by moving the N_c'' D4-branes with $NS5'_R$ -brane to $+v$ direction in the electric theory and then consider the Seiberg dual for the second gauge group in this subsection ³.

- Mass deformation by G and \tilde{G}

There is no electric superpotential in Figure 2A. It is known in [20] that the classical superpotential without $NS5'_R$ -brane (and its mirror) for the particular orientation for NS-branes in Figure 2A is vanishing. Now we are adding the $NS5'_R$ -brane (and its mirrors) into the brane configuration of [20] together with N_c'' D4-branes (and their mirrors). Since the angle between the NS5-brane and the extra $NS5'_R$ -brane is $\frac{\pi}{2}$ in Figure 2A, the mass for the extra adjoint field μ'' corresponding to the gauge group $SU(N_c'')$ becomes large and after integrating out this adjoint field, then the extra classical superpotential term which has a factor $\frac{1}{\mu''}$ will vanish.

Now let us deform this theory. Displacing the two NS5'-branes relative each other in the $+v$ direction corresponds to turning on a quadratic mass-deformed electric superpotential for the field G as follows:

$$W_{def} = mG\tilde{G}(\equiv m \text{tr } \Phi''), \quad m = \frac{\Delta x}{\ell_s^2} \quad (2.2)$$

where the second gauge group indices in G and \tilde{G} are contracted, the third gauge group indices in them are free and the mass m is related to the relative displacement Δx between the two NS5'-branes where $x \equiv x^5$. The gauge-singlet Φ'' , which has two free indices on the third gauge group $SU(N_c'')$, is in the adjoint representation for the third gauge group $SU(N_c'')$, i.e., $(\mathbf{1}, \mathbf{1}, \mathbf{N}_c''^2 - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the gauge group. The trace in (2.2) is taken over the third gauge group indices and the gauge singlet Φ'' is a $N_c'' \times N_c''$ matrix. The $NS5'_R$ -brane together with N_c'' -color D4-branes is moving to the $+v$ direction for fixed other branes

³When we move NS-branes or D4-branes, we only refer to the original NS-branes or D4-branes located at the positive x^6 region. The movement of their mirrors is evident if we remember the action of $O6^+$ -plane we mentioned before. Then, although we describe only the movement of original NS-branes or D4-branes, their mirrors also are moving automatically, according to the action of $O6^+$ -plane.

during this particular mass deformation (and their mirrors to $-v$ direction). Then the x^5 coordinate of $NS5'_L$ -brane is equal to zero and the x^5 coordinate of $NS5'_R$ -branes is given by Δx . Giving an expectation value to the meson field Φ'' corresponds to recombination of N'_c - and N''_c - color D4-branes, which will become N''_c - or N'_c -color D4-branes in Figure 2A such that they are suspended between the $NS5'_L$ -brane and the $NS5'_R$ -brane and moving them into the $w \equiv x^8 + ix^9$ direction. We assume that the number of colors satisfies

$$N_c + N''_c \geq N'_c \geq N_c. \quad (2.3)$$

Now we draw this $\mathcal{N} = 1$ supersymmetric electric brane configuration in Figure 2B for nonvanishing mass for the fields G and \tilde{G} . The geometric configuration for three NS-branes in Figure 2B is exactly the same as the first three NS-branes in Figure 1B of [17].

- Seiberg dual

Now it is ready to take the magnetic gauge theory from the above specific deformed electric gauge theory. This is a necessary step for the construction of meta-stable brane configuration. By applying the Seiberg dual to the second gauge group $SU(N'_c)$ factor in (2.1), the $NS5'_{L,R}$ -branes can be placed at the outside of the three NS5-branes, as in Figure 3. Starting from Figure 2B and interchanging the $NS5'_L$ -brane and the NS5-brane (and their mirrors), one obtains the Figure 3A with D4- and $\overline{D4}$ -branes.

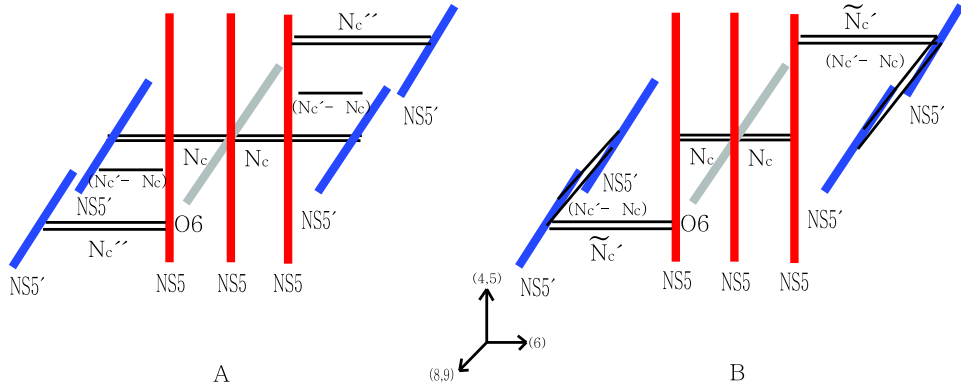


Figure 3: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(N_c) \times SU(\tilde{N}'_c = N_c + N''_c - N'_c) \times SU(N''_c)$ corresponding to Figure 2B with D4- and $\overline{D4}$ -branes (3A) and with a misalignment between D4-branes (3B) when the NS5'-branes of 3A are close to each other. The number of tilted D4-branes in 3B can be written as $N'_c - N_c = N''_c - \tilde{N}'_c$. The notation for the anti-D4-branes is used with the bar on the number of those branes in Figure 3A.

Before arriving at the Figure 3A, there exists an intermediate step where the $(N''_c - N'_c + N_c)$ D4-branes are connecting between the NS5-brane and the $NS5'_L$ -brane, N''_c D4-branes are connecting between the $NS5'_L$ -brane and $NS5'_R$ -brane (and their mirrors) as well

as N_c D4-branes between the NS5-brane and its mirror. By reconnecting the N_c'' D4-branes connecting between the NS5-brane and the $NS5'_L$ -brane with the N_c'' D4-branes connecting between $NS5'_L$ -brane and the $NS5'_R$ -brane and moving those combined N_c'' D4-branes to $+v$ -direction (and their mirrors to $-v$ direction), one gets the final Figure 3A where we are left with $(N'_c - N_c)$ anti-D4-branes between the NS5-brane and $NS5'_L$ -brane.

When two NS5'-branes in Figure 3A are close to each other, one arrives at the Figure 3B by realizing that the number of N_c'' D4-branes connecting between NS5-brane and $NS5'_R$ -brane in Figure 3A can be rewritten as $(N'_c - N_c)$ plus \tilde{N}'_c . The brane configuration consisting of NS5-brane and two NS5'-branes in Figure 3B is exactly the same as the first three NS-branes in Figure 3B of [17]. Moreover if we change the $O6^+$ -plane into the $O6^-$ -plane, replace the number of color N_c by $2N_c$, and remove the middle NS5-brane in Figure 3, then this is exactly the same as the Figure 12 of [17].

The dual gauge group from the computation of linking number counting during the brane motion is given by

$$SU(N_c) \times SU(\tilde{N}'_c = N_c + N_c'' - N'_c) \times SU(N_c''). \quad (2.4)$$

The matter contents in the magnetic theory are a symmetric tensor field S charged under $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + 1), \mathbf{1}, \mathbf{1})$, the field f charged under $(\mathbf{N}_c, \overline{\tilde{\mathbf{N}}}'_c, \mathbf{1})$, a field g charged under $(\mathbf{1}, \tilde{\mathbf{N}}'_c, \overline{\mathbf{N}}_c'')$, and their conjugates \tilde{S}, \tilde{f} and \tilde{g} under the dual gauge group (2.4) and the gauge-singlet Φ'' for the second dual gauge group in the adjoint representation for the third dual gauge group, i.e., $(\mathbf{1}, \mathbf{1}, \mathbf{N}_c''^2 - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the dual gauge group. Then the gauge singlet Φ'' which was defined in (2.2) is nothing but a $N_c'' \times N_c''$ matrix.

- Meta-stable brane configuration

The combination of cubic superpotential in the magnetic theory with the deformed mass term (2.2) is given by ⁴

$$W_{dual} = \Phi'' g \tilde{g} + m \text{tr} \Phi''. \quad (2.5)$$

Here the magnetic fields g and \tilde{g} correspond to 4-4 strings connecting the \tilde{N}'_c -color D4-branes that are connecting between the NS5-brane and the $NS5'_R$ -brane in Figure 3B with N_c'' -flavor D4-branes which are realized as corresponding D4-branes in Figure 3A. Although the N_c'' D4-branes in Figure 3A cannot move any directions, the tilted $(N'_c - N_c)$ -flavor D4-branes can move w direction in Figure 3B. The remaining upper \tilde{N}'_c D4-branes are fixed also and cannot

⁴In general, there are also the extra terms in the superpotential $\Phi_1 f g + \Phi f \tilde{f} + \Phi_3 \tilde{f} \tilde{g}$ where we define $\Phi_1 \equiv FG$, $\Phi \equiv F\tilde{F}$ and $\Phi_3 \equiv \tilde{F}\tilde{G}$, coming from different bifundamentals. However, the F- term conditions, $\Phi_1 g + \Phi f = 0 = \Phi f + \Phi_3 \tilde{g}$ lead to $\langle \Phi_1 \rangle = \langle \Phi_3 \rangle = \langle f \rangle = \langle \tilde{f} \rangle = 0$. Therefore, these extra terms do not contribute to the one loop computation up to quadratic order.

move any directions. It is useful for the transition from Figure 3A to Figure 3B to note that there is a decomposition

$$N_c'' = (N_c' - N_c) + \tilde{N}_c'.$$

The N_c D4-branes between the NS5-brane and the middle NS5-brane can slide along the v direction (and their mirrors to the opposite direction).

The brane configuration for zero mass for the bifundamental G and \tilde{G} , which has only a cubic superpotential in (2.5), can be seen from the Figure 3A by moving the upper NS5'-brane (or $NS5'_R$ -brane) together with N_c'' color D4-branes into the origin $v = 0$ (and their mirrors): massless limit after the Seiberg dual with massive case. Then the number of dual colors for D4-branes becomes N_c between the NS5-brane and its mirror, \tilde{N}_c' between the NS5-brane and the $NS5'_L$ -brane and N_c'' between the $NS5'_L$ -brane and the $NS5'_R$ -brane. Or starting from the Figure 2A and moving the NS5-brane to the left all the way past the $NS5'_L$ -brane (and their mirrors), one also obtains the corresponding magnetic brane configuration for massless case: the Seiberg dual with massless case.

The brane configuration in Figure 3A is stable as long as the distance Δx between the upper NS5'-brane and the lower NS5'-brane is large. The fundamental strings are stretched between D4 and $\overline{D4}$ -branes in Figure 3A and the lowest exciting states are given by open string tachyon. Now the effective mass for open string tachyon can be computed via near horizon geometry of the NS5-branes and is given by the result (3.16) of [8]. Then for $\Delta x > \pi l_s$, the open string tachyon is massive everywhere and the Figure 3A is locally stable and global ground state of the system in this regime at least classically.

If they are close to each other, then this brane configuration of Figure 3A is unstable to decay and leads to the meta-stable brane configuration in Figure 3B. One regards these two type IIA brane configurations, in the zero limit of g_s , as particular states in the magnetic gauge theory, when Δx or m is very small, with the gauge group (2.4) and superpotential (2.5), along the line of [8].

One can perform similar analysis done in [8, 9, 17] for our brane configuration since one can take into account the behavior of parameters geometrically in the presence of $O6^+$ -plane. Then the upper tilted $(N_c'' - \tilde{N}_c')$ flavor D4-branes of straight brane configuration of Figure 3B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and the NS5-brane from the DBI action as long as y_1 , the x^6 coordinate of NS5-brane, is very large. Then the mirror of NS5-brane and the middle NS5-brane do not affect those flavor D4-branes. On the other hand, if y_1 goes to zero, then the mirror of NS5-brane plus the middle NS5-brane play the role of enhancing the strength for the NS5-branes

and will affect both the energy of bending curve, E_{curved} , and Δx as in [8]. Of course, their mirrors, the lower tilted $(N_c'' - \tilde{N}_c')$ flavor D4-branes of straight brane configuration of Figure 3B can bend and their trajectories connecting two NS5'-branes should be preserved under the $O6^+$ -plane action.

- Gauge theory at small Δx

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (2.4) and the gauge couplings for the three gauge group factors are given by

$$g_{1,mag}^2 = \frac{g_s \ell_s}{(y_1 + y_2)}, \quad g_{2,mag}^2 = \frac{g_s \ell_s}{y_2}, \quad g_{3,mag}^2 = \frac{g_s \ell_s}{(y_3 + y_2)}$$

where y_1 is the x^6 coordinate of $NS5'_L$ -brane, $y_1 + y_2$ is the x^6 coordinate of NS5-brane, and $y_1 + y_2 + y_3$ is the x^6 coordinate of $NS5'_R$ -brane in Figure 3, as we described before. The dual gauge theory has a meson Φ'' and bifundamentals f, \tilde{f}, g and \tilde{g} as well as a symmetric tensor S and its conjugate \tilde{S} under the dual gauge group (2.4) and the superpotential corresponding to the Figures 3A and 3B is given by

$$W_{dual} = h\Phi''g\tilde{g} - h\mu^2 \text{tr} \Phi'', \quad h^2 = g_{3,mag}^2, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}. \quad (2.6)$$

Then the product $g\tilde{g}$ is a $\tilde{N}_c' \times \tilde{N}_c'$ matrix where the third gauge group indices for g and \tilde{g} are contracted with those of Φ'' and Φ'' is a $N_c'' \times N_c''$ matrix. The product $g\tilde{g}$ has the same representation for the product of quarks and the third gauge group indices for the field Φ'' play the role of the flavor indices.

When the upper NS5'-brane(or $NS5'_R$ -brane) is replaced by coincident N_c'' D6-branes in Figure 3B, this brane configuration looks similar to the Figure 5 found in [20] where the gauge group was given by $SU(n_c) \times SU(n'_f + n_c - n'_c)$ with n'_f multiplets, bifundamentals, a symmetric flavor, a conjugate symmetric flavor and various gauge singlets. Then the present N_c for the first gauge group factor corresponds to the n_c , the N_c' for the second gauge group factor corresponds to n'_c , and finally the N_c'' corresponds to the n'_f of [20]. The location of $NS5'_R$ -brane of Figure 3B is in the right hand side of NS5-brane while the location of n'_f D6-branes of Figure 5 in [20] is in the left hand side of NS5-brane.

When y_1 goes to zero, this meta-stable brane configuration of Figure 3B where there are three NS5-branes at the origin reduces to the Figure 4 of [9] where there is a single NS5-brane at the origin.

Therefore, the F-term equation, the derivative of W_{dual} in (2.6) with respect to the meson field Φ'' cannot be satisfied if the N_c'' exceeds \tilde{N}_c' . So the supersymmetry is broken. That is,

there exist three equations from F-term conditions: $g^a \tilde{g}_b - \mu^2 \delta_b^a = 0$ and $\Phi'' g = 0 = \tilde{g} \Phi''$. Then the solutions for these, as usual, are given by

$$\langle g \rangle = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \quad \langle \tilde{g} \rangle = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \quad \langle \Phi'' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi''_0 \mathbf{1}_{(N''_c - \tilde{N}'_c)} \end{pmatrix} \quad (2.7)$$

where the zero of $\langle g \rangle$ is a $(N''_c - \tilde{N}'_c) \times \tilde{N}'_c$ matrix, the zero of $\langle \tilde{g} \rangle$ is a $\tilde{N}'_c \times (N''_c - \tilde{N}'_c)$ matrix and the zeros of $\langle \Phi'' \rangle$ are $\tilde{N}'_c \times \tilde{N}'_c$, $\tilde{N}'_c \times (N''_c - \tilde{N}'_c)$ and $(N''_c - \tilde{N}'_c) \times \tilde{N}'_c$ matrices. Then one can expand these fields around on a point (2.7), as in [1, 23] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{eff}^{(1)}$ for Φ''_0 leads to the positive value for $m_{\Phi''_0}^2$ implying that these vacua are stable. The gauge theory analysis where the theory will be strongly coupled in the IR region is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C .

2.2 $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c) \times SU(\tilde{N}''_c)$ magnetic theory

Let us take the mass deformation by moving the $(N'_c - N_c)$ D4-branes suspending between the $NS5'_L$ -brane and the NS5-brane to $+v$ direction in the electric theory and then describe the Seiberg dual for the third gauge group.

- Mass deformation by G and \tilde{G}

Let us consider other magnetic theory for the same undeformed electric theory (characterized by Figure 4A which is the same as Figure 2A) given in the subsection 2.1 by considering the different deformation in electric theory (characterized by Figure 4B). Displacing the two NS5'-branes relative each other in the $+v$ direction, as before, corresponds to turning on a quadratic mass-deformed electric superpotential for the field G with different contractions as follows:

$$W_{def} = m G \tilde{G} (\equiv m \text{tr } \Phi'), \quad m = \frac{\Delta x}{\ell_s^2} \quad (2.8)$$

where the third gauge group indices in G and \tilde{G} are contracted and the mass m is related to the relative displacement Δx between two NS5'-branes. The gauge-singlet Φ' , which has two free indices on the gauge group $SU(N'_c)$, is in the adjoint representation for the second gauge group $SU(N'_c)$, i.e., $(\mathbf{1}, (\mathbf{N}'_c - \mathbf{N}_c)^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the gauge group where the gauge group $SU(N'_c)$ is broken to $SU(N'_c - N_c)$. That is, the gauge singlet Φ' is a $(N'_c - N_c) \times (N'_c - N_c)$ matrix. The $NS5'_L$ -brane together with $(N'_c - N_c)$ -color D4-branes is moving to the $+v$ direction for fixed other branes during this particular mass deformation (and their mirrors to $-v$ direction). Then the x^5 coordinate of $NS5'_R$ -brane is equal to zero and the x^5 coordinate of $NS5'_L$ -branes is given by Δx . Giving an expectation value to the meson field Φ' corresponds

to recombination of N'_c - and N''_c - color D4-branes, which will become N''_c -color D4-branes in Figure 4A such that they are suspending between the $NS5'_L$ -brane and the $NS5'_R$ -brane and moving them into the w direction. We assume, as before, that the number of colors satisfies

$$N'_c \geq N''_c \geq N_c.$$

Now we draw this $\mathcal{N} = 1$ supersymmetric electric brane configuration in Figure 4B for nonvanishing mass for the fields G and \tilde{G} . The geometric configuration for three NS-branes in Figure 4B is exactly the same as the first three NS-branes in Figure 3B of [17].

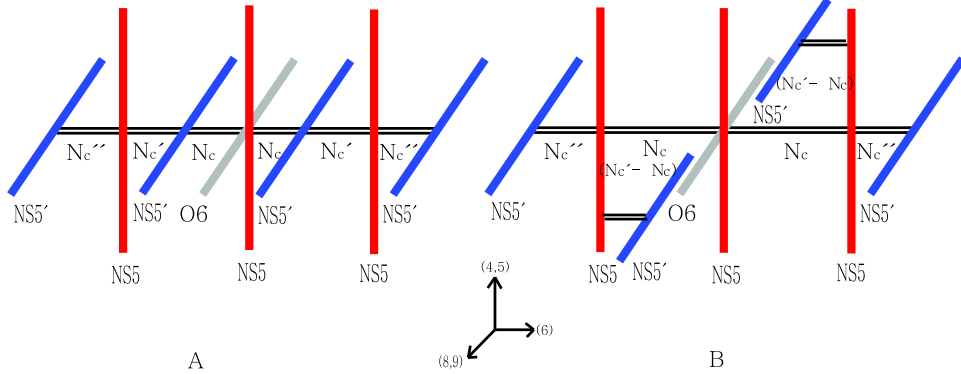


Figure 4: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ and a symmetric tensor S , a conjugate symmetric tensor \tilde{S} , bifundamentals F, \tilde{F}, G and \tilde{G} with vanishing(4A) which is identical to Figure 1A and non-vanishing(4B) mass for the bifundamentals G and \tilde{G} . In 4B, the $NS5'_L$ -brane together with $(N'_c - N_c)$ D4-branes is moving to $+v$ direction (and their mirrors to $-v$ direction). The remaining N_c D4-branes are reconnecting with those D4-branes in 4A and finally combined those D4-branes are suspending between NS5-brane and its mirror in 4B.

- Seiberg dual

By applying the Seiberg dual to the third gauge group $SU(N''_c)$ factor in (2.1), the $NS5'_{L,R}$ -branes can be located at the inside of the two outer NS5-branes, as in Figure 5. Starting from Figure 4B and interchanging the NS5-brane and the $NS5'_R$ -brane (and their mirrors), one obtains the Figure 5A with D4- and $\overline{D4}$ -branes. Before arriving at the Figure 5A, there exists an intermediate step where the $(N'_c - N_c)$ D4-branes are connecting between the $NS5'_L$ -brane and the $NS5'_R$ -brane, $(N'_c - N''_c)$ D4-branes are connecting between the $NS5'_R$ -brane and NS5-brane (and their mirrors) as well as N_c D4-branes between $NS5'_R$ -brane and its mirror. By reconnecting the $(N'_c - N_c)$ D4-branes connecting between the $NS5'_L$ -brane and the $NS5'_R$ -brane with the $(N'_c - N_c)$ D4-branes connecting between $NS5'_R$ -brane and the NS5-brane where we introduce $-N_c$ D4-branes and $-N_c$ anti D4-branes and moving those combined D4-

branes to $+v$ -direction (and their mirrors to $-v$ direction), one gets the final Figure 5A where we are left with $(N_c'' - N_c)$ anti-D4-branes between the $NS5'_R$ -brane and the NS5-brane.

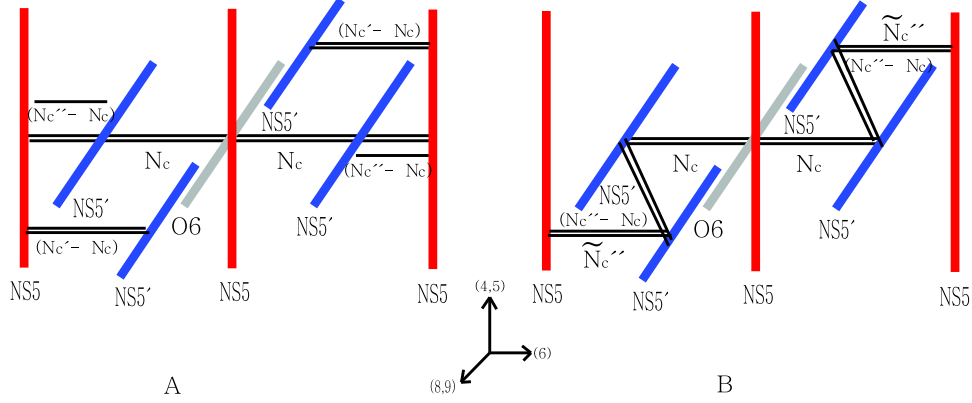


Figure 5: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'' = N'_c - N_c'')$ with D4- and $\overline{D4}$ -branes(5A) and with a misalignment between D4-branes(5B) when the NS5'-branes of 5A are close to each other. The number of tilted D4-branes can be written as $N_c'' - N_c = (N'_c - N_c) - \tilde{N}_c''$ in 5B.

When two NS5'-branes in Figure 5A are close to each other, then it leads to Figure 5B by realizing that the number of $(N'_c - N_c)$ D4-branes connecting between $NS5'_L$ -brane and NS5-brane in Figure 5A can be rewritten as $(N_c'' - N_c)$ plus \tilde{N}_c'' . The brane configuration consisting of NS5-brane and two NS5'-branes in Figure 5B is exactly the same as those in Figure 4B of [17]. Moreover if we change the $O6^+$ -plane into the $O6^-$ -plane, replace the number of color N_c by $2N_c$, and remove the middle NS5-brane in Figure 5, then this is exactly the same as the Figure 14 of [17].

The dual gauge group can be read off and is given by

$$SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'' = N'_c - N_c''). \quad (2.9)$$

The matter contents are a symmetric tensor field S charged under $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + 1), \mathbf{1}, \mathbf{1})$, the field F charged under $(\mathbf{N}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$, a field g charged under $(\mathbf{1}, \mathbf{N}'_c, \tilde{\mathbf{N}}_c'')$ and their conjugates \tilde{S}, \tilde{F} and \tilde{g} under the dual gauge group (2.9) and the gauge-singlet Φ' which is in the adjoint representation for the second dual gauge group, in other words, $(\mathbf{1}, (\mathbf{N}'_c - \mathbf{N}_c)^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the dual gauge group (2.9) where the gauge group $SU(N'_c)$ is broken to $SU(N'_c - N_c)$. Then the Φ' is a $(N'_c - N_c) \times (N'_c - N_c)$ matrix. Only $(N'_c - N_c)$ D4-branes among N'_c D4-branes can participate in the mass deformation we are considering.

- Meta-stable brane configuration

The cubic superpotential plus the mass term in the magnetic theory is given by

$$W_{dual} = \Phi' g \tilde{g} + m \text{tr} \Phi' \quad (2.10)$$

where Φ' was defined as $\Phi' \equiv G\tilde{G}$ from (2.8) and the third gauge group indices in G and \tilde{G} are contracted and each second gauge group index in them is encoded in Φ' . Although the Φ' that has second gauge group indices looks similar to the previous Φ'' that has third gauge group indices, their group indices are different from each other. Here the magnetic fields g and \tilde{g} correspond to 4-4 strings connecting the \tilde{N}_c'' -color D4-branes(that are connecting between the $NS5'_L$ -brane and the NS5-brane in Figure 5B) with N'_c -flavor D4-branes. Among these N'_c -flavor D4-branes, only the strings ending on the upper $(N'_c - N_c'')$ D4-branes and on the tilted $(N_c'' - N_c)$ D4-branes in Figure 5B enter the cubic superpotential term. Note that the summation of these D4-branes is equal to $(N'_c - N_c)$. Although the $(N'_c - N_c)$ D4-branes in Figure 5A cannot move any directions, the tilted $(N_c'' - N_c)$ -flavor D4-branes can move w direction. The remaining upper \tilde{N}_c'' D4-branes are fixed also and cannot move any direction. As before, it is useful to understand the transition from Figure 5A to Figure 5B to note that there is a decomposition

$$(N'_c - N_c) = (N_c'' - N_c) + \tilde{N}_c''.$$

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential (2.10), can be obtained from Figure 5A by moving the upper NS5'-brane together with $(N'_c - N_c)$ color D4-branes into the origin $v = 0$ (and their mirrors). Then the number of dual colors for D4-branes becomes N_c between the $NS5'_L$ -brane and its mirror, N'_c between the $NS5'_L$ -brane and the $NS5'_R$ -brane and \tilde{N}_c'' between $NS5'_R$ -brane and NS5-brane. Or starting from Figure 4A and moving the NS5-brane to the right all the way past the $NS5'_R$ -brane(and their mirrors), one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 5A is stable as long as the distance Δx between the upper NS5'-brane and the lower NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay to the meta-stable brane configuration in Figure 5B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.9) and superpotential (2.10). Then the upper tilted $(N'_c - N_c - \tilde{N}_c'')$ flavor D4-branes of straight brane configuration of Figure 5B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane, as long as y_1 is very large. Of course, their mirrors, the lower tilted $(N'_c - N_c - \tilde{N}_c'')$ flavor D4-branes of straight brane configuration of Figure 5B can bend and their trajectories connecting two NS5'-branes should be preserved under the $O6^+$ -plane action. The N_c D4-branes between the $NS5'_R$ -brane and its mirror can slide along the w direction.

When the upper NS5'-brane(or $NS5'_L$ -brane) is replaced by coincident $(N'_c - N_c)$ D6-

branes in Figure 5B, this brane configuration looks similar to the Figure 7 found in [20]. The only difference between these two brane configurations appears the brane contents at the origin $x^6 = 0$. The gauge group in [20] was given by $SU(n_c) \times SU(n'_f + n_c - n'_c)$ with n'_f fundamentals, bifundamentals, an antisymmetric flavor, a conjugate symmetric flavor, and various gauge singlets where there are NS5'-brane, $O6^\pm$ -planes, and eight semi infinite D6-branes at $x^6 = 0$. Then the our N_c corresponds to the n_c , the number $(N'_c - N_c)$ corresponds to n'_f , and moreover N''_c corresponds to the n'_c of [20].

- Gauge theory at small Δx

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (2.9) and the gauge couplings for the three gauge group factors are given by similarly. The dual gauge theory has a meson Φ' and bifundamentals f, \tilde{f}, g and \tilde{g} as well as a symmetric tensor S and its conjugate \tilde{S} under the dual gauge group (2.9) and the superpotential corresponding to the Figures 5A and 5B is given by

$$W_{dual} = h\Phi'g\tilde{g} - h\mu^2 \text{tr} \Phi', \quad h^2 = g_{2,mag}^2. \quad (2.11)$$

Then $g\tilde{g}$ is a $\tilde{N}''_c \times \tilde{N}''_c$ matrix where the second gauge group indices for g and \tilde{g} are contracted with those of Φ' while the Φ' is a $(N'_c - N_c) \times (N'_c - N_c)$ matrix and the μ^2 is the same as before. The product $g\tilde{g}$ has the same representation for the product of quarks and the second gauge group indices for the field Φ' play the role of the flavor indices, as before.

Therefore, the F-term equation, the derivative of W_{dual} (2.11) with respect to the meson field Φ' cannot be satisfied if the $(N'_c - N_c)$ exceeds \tilde{N}''_c . So the supersymmetry is broken. That is, there exist three equations from F-term conditions: $g^a \tilde{g}_b - \mu^2 \delta_b^a = 0$ and $\Phi'g = 0 = \tilde{g}\Phi'$. Then the solutions for these are given by

$$\langle g \rangle = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}''_c} \\ 0 \end{pmatrix}, \quad \langle \tilde{g} \rangle = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}''_c} & 0 \end{pmatrix}, \quad \langle \Phi' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi'_0 \mathbf{1}_{(N'_c - N_c) - \tilde{N}''_c} \end{pmatrix} \quad (2.12)$$

where the zero of $\langle g \rangle$ is a $(N'_c - N_c - \tilde{N}''_c) \times \tilde{N}''_c$ matrix, the zero of $\langle \tilde{g} \rangle$ is a $\tilde{N}''_c \times (N'_c - N_c - \tilde{N}''_c)$ matrix and the zeros of $\langle \Phi' \rangle$ are $\tilde{N}''_c \times \tilde{N}''_c$, $\tilde{N}''_c \times (N'_c - N_c - \tilde{N}''_c)$ and $(N'_c - N_c - \tilde{N}''_c) \times \tilde{N}''_c$ matrices. Then one can expand these fields around on a point (2.12), as in [1, 23] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{eff}^{(1)}$ for Φ'_0 leads to the positive value for $m_{\Phi'_0}^2$ implying that these vacua are stable. The gauge theory analysis is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C .

2.3 $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(N'_c) \times SU(N''_c)$ magnetic theory

Let us consider the Seiberg dual for the first gauge group and take the mass deformation by moving the $(N'_c - N''_c)$ D4-branes between the middle NS5-brane and the $NS5'_M$ -brane to $+v$ direction.

- Seiberg dual and mass deformation

Starting from the Figure 4A, we apply the Seiberg dual to the first gauge group $SU(N_c)$ factor and the $NS5'_L$ -brane and its mirror are interchanged each other. Then the brane configuration⁵ is the same as Figure 4A except that the number of color \tilde{N}_c is given by $\tilde{N}_c = 2N'_c - N_c$ from [20, 14]. By rotating the NS5-brane by $\frac{\pi}{2}$ (leading to $NS5'_M$ -brane) and moving it together with $(N'_c - N''_c)$ D4-branes to $+v$ direction, then the $(N'_c - N''_c)$ D4-branes are connecting between the $NS5'_L$ -brane and the $NS5'_M$ -brane and \tilde{N}_c D4-branes connecting between the middle NS5-brane and $NS5'_L$ -brane as well as N''_c D4-branes between the $NS5'_L$ -brane and the $NS5'_R$ -brane (and their mirrors).

By introducing $(N'_c - N''_c)$ D4-branes and $(N'_c - N''_c)$ anti-D4-branes between the middle NS5-brane and $NS5'_L$ -brane, reconnecting the former with the $(N'_c - N''_c)$ D4-branes connecting between the $NS5'_L$ -brane and the $NS5'_M$ -brane and moving those combined D4-branes to $+v$ -direction (and their mirrors to $-v$ direction), one gets the final Figure 6A where we are left with $(N'_c - N''_c - \tilde{N}_c)$ anti-D4-branes between the middle NS5-brane and $NS5'_L$ -brane.

Now we draw this $\mathcal{N} = 1$ supersymmetric magnetic brane configuration in Figure 6. We assume, as before, that the number of colors satisfies

$$2N'_c \geq N_c \geq N'_c + N''_c.$$

When two NS5'-branes in Figure 6A are close to each other, then it leads to Figure 6B by realizing that the number of $(N'_c - N''_c)$ D4-branes connecting between the NS5-brane and the $NS5'_M$ -brane in Figure 6A can be rewritten as $(N_c - N'_c - N''_c)$ plus \tilde{N}_c . The brane configuration consisting of the middle NS5-brane and first two NS5'-branes in Figure 6B is exactly the same as the first three NS-branes in Figure 2B of [17].

The dual gauge group is given by

$$SU(\tilde{N}_c = 2N'_c - N_c) \times SU(N'_c) \times SU(N''_c) \quad (2.13)$$

where the number of dual color can be obtained from the linking number counting, as done in [20, 14]. Note that the number of flavors of D6-branes in [20] is zero because in the present

⁵This duality symmetry in field theory side was tested in [11] and the brane configuration in type IIA string theory for the supersymmetric magnetic theory was studied in [21]. Moreover, the nonsupersymmetric meta-stable brane configuration was found in [14] where the correct movement of the branes were crucial.

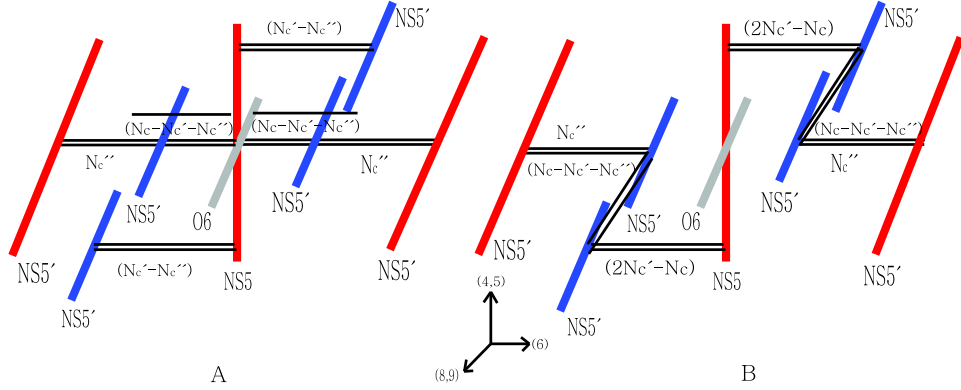


Figure 6: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(\tilde{N}_c = 2N'_c - N_c) \times SU(N'_c) \times SU(N''_c)$ with D4- and $\overline{D4}$ -branes(6A) and with a misalignment between D4-branes(6B) when the NS5'-branes are close to each other. The number of tilted D4-branes in 6B can be written as $N_c - N'_c - N''_c = (N'_c - N''_c) - \tilde{N}_c$.

case, there are no D6-branes. The matter contents are the flavor singlet f in the bifundamental representation $(\tilde{N}_c, \overline{N'_c}, \mathbf{1})$ and its complex conjugate field \tilde{f} in the bifundamental representation $(\overline{\tilde{N}_c}, N'_c, \mathbf{1})$, under the dual gauge group (2.13) and the gauge singlet Φ' in the representation for $(\mathbf{1}, (N'_c - N''_c)^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the dual gauge group where the gauge group $SU(N'_c)$ is broken to $SU(N'_c - N''_c)$. There are also the symmetric flavor s for $SU(\tilde{N}_c)$ and the conjugate symmetric flavor \tilde{s} for $SU(\tilde{N}_c)$ as well as G and \tilde{G} . Then the Φ' is a $(N'_c - N''_c) \times (N'_c - N''_c)$ matrix. Only $(N'_c - N''_c)$ D4-branes among N'_c D4-branes can participate in the mass deformation. A cubic superpotential is an interaction between dual “quarks” and a meson.

- Meta-stable brane configuration

Then the dual magnetic superpotential, by adding the mass term for the bifundamental F which can be interpreted as a linear term in the meson Φ' to this cubic superpotential, is given by

$$W_{dual} = \Phi' f \tilde{f} + m \text{tr} \Phi', \quad \Phi' \equiv F \tilde{F}, \quad m = \frac{\Delta x}{\ell_s^2} \quad (2.14)$$

where Φ' was defined as $\Phi' \equiv F \tilde{F}$ and the first gauge group indices in F and \tilde{F} are contracted and each second gauge group index in them is encoded in Φ' .

Here the magnetic fields f and \tilde{f} correspond to 4-4 strings connecting the \tilde{N}_c -color D4-branes(that are connecting between the NS5-brane and the $NS5'_M$ -brane in Figure 6B) with N'_c -flavor D4-branes. Among these N'_c -flavor D4-branes, only the strings ending on the upper $(2N'_c - N_c)$ D4-branes and on the tilted $(N_c - N'_c - N''_c)$ D4-branes in Figure 6B enter the cubic superpotential term. Note that the summation of these D4-branes is equal to $(N'_c - N''_c)$.

Although the $(N'_c - N''_c)$ D4-branes in Figure 6A cannot move any directions, the tilted $(N_c - N'_c - N''_c)$ -flavor D4-branes can move w direction. The remaining upper \tilde{N}_c D4-branes are fixed also and cannot move any direction. It is useful to understand the transition from Figure 6A to Figure 6B to note that there is a decomposition

$$(N'_c - N''_c) = (N_c - N'_c - N''_c) + \tilde{N}_c.$$

The brane configuration for zero mass for the bifundamentals can be obtained from Figure 6A by pushing the $NS5'_M$ -brane with $(N'_c - N''_c)$ D4-branes into the origin $v = 0$. Then the number of dual colors for D4-branes becomes \tilde{N}_c between the NS5-brane and the $NS5'_L$ -brane, N'_c between the $NS5'_L$ -branes and the $NS5'_M$ -brane, and N''_c between the $NS5'_M$ -brane and the $NS5'_R$ -brane.

The brane configuration in Figure 6A is stable as long as the distance Δx between the upper $NS5'_M$ -brane and the middle $NS5'_L$ -brane is large. If they are close to each other, then this brane configuration is unstable to decay and leads to the meta-stable brane configuration in Figure 6B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.13) and superpotential (2.14).

When the $NS5'_M$ -brane which is connected by \tilde{N}_c D4-branes is replaced by $(N'_c - N''_c)$ D6-branes, the brane configuration of Figure 6B together with the rotation of $NS5'_R$ -brane by $\frac{\pi}{2}$ is the same as the Figure 4 (with the movement of n_f D6-branes to the opposite v direction) studied in [20] where the gauge group is given by $SU(2n_f + 2n'_c - n_c) \times SU(n'_c)$ with n_f fundamentals, bifundamentals, a symmetric flavor, a conjugate symmetric flavor, and various gauge singlets. Then the our N_c corresponds to the n_c , the number $(N'_c - N''_c)$ corresponds to n_f , and our N''_c corresponds to the n'_c of [20].

One can perform similar analysis in our brane configuration since one can take into account the behavior of parameters geometrically in the presence of $O6^+$ -plane. Then the upper tilted $(N'_c - N''_c - \tilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 6B bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action. Of course, their mirrors, the lower $(N'_c - N''_c - \tilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 6B bend and their trajectory connecting two NS5'-branes should be preserved under the $O6^+$ -plane action. The correct choice for the ground state of the system depends on the parameters y_i and Δx .

The N''_c D4-branes between the $NS5'_L$ -brane and the $NS5'_R$ -brane can slide along the w direction.

- Gauge theory at small Δx

The low energy dynamics of the magnetic brane configuration can be described by the

$\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (2.13) and the gauge couplings for the gauge group factors are given by similarly. The dual gauge theory has a meson field Φ' and bifundamental f in the representation $(\tilde{\mathbf{N}}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$ under the dual gauge group (2.13) and the superpotential corresponding to the Figures 6A and 6B is given by

$$W_{dual} = h\Phi' f \tilde{f} - h\mu^2 \text{tr } \Phi', \quad h^2 = g_{2,mag}^2$$

and the mass parameter μ^2 is the same as before. See also the relevant papers [14, 9]. Then $f\tilde{f}$ is a $\tilde{N}_c \times \tilde{N}_c$ matrix where the second gauge group indices for f and \tilde{f} are contracted with those of Φ' while Φ' is a $(N'_c - N''_c) \times (N'_c - N''_c)$ matrix. Although the field f itself is an antifundamental in the second gauge group which is a different feature, compared with the singlet representation for the usual quark coming from D6-branes [20], the product $f\tilde{f}$ has the same representation with the product of quarks. Moreover, the second gauge group indices for the field Φ' play the role of the flavor indices.

Therefore, the F-term equation, the derivative of W_{dual} with respect to the meson field Φ' cannot be satisfied if the $(N'_c - N''_c)$ exceeds \tilde{N}_c . So the supersymmetry is broken. That is, there are three equations from F-term conditions: $f^a \tilde{f}_b - \mu^2 \delta_b^a = 0$, $\Phi' f = 0$, and $\tilde{f} \Phi' = 0$. Then the solutions for these are given by

$$\langle f \rangle = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}_c} \\ 0 \end{pmatrix}, \quad \langle \tilde{f} \rangle = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}_c} & 0 \end{pmatrix}, \quad \langle \Phi' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi'_0 \mathbf{1}_{(N'_c - N''_c - \tilde{N}_c)} \end{pmatrix}.$$

At one loop, the effective potential $V_{eff}^{(1)}$ for Φ'_0 leads to the positive value for $m_{\Phi'_0}^2$ implying that these vacua are stable. The gauge theory analysis where the theory will be strongly coupled in the IR region $N'_c - N''_c > 2\tilde{N}_c - 2$ is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C , as done in [9].

3 Meta-stable brane configurations-II

In this section, we continue to construct the meta-stable brane configurations for the same gauge theory we have considered in previous section but different matter contents. In the brane configuration context, the roles of NS5-brane and NS5'-brane are interchanged in an electric brane configuration and the brane contents at the origin $x^6 = 0$ are different from the previous case as we will see later.

The type IIA brane configuration corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group

$$SU(N_c) \times SU(N'_c) \times SU(N''_c) \tag{3.1}$$

and with an antisymmetric tensor field A charged under $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c - \mathbf{1}), \mathbf{1}, \mathbf{1})$, a conjugate symmetric tensor field \tilde{S} charged under $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1}, \mathbf{1})$, an eight fundamentals \hat{Q} charged under $(\mathbf{N}_c, \mathbf{1}, \mathbf{1})$, a field F charged under $(\mathbf{N}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$, a field G charged under $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}}''_c)$, and their conjugates \tilde{F} and \tilde{G} can be described by the left $NS5_L$ -brane, the NS5'-brane, the right $NS5_R$ -brane (and their mirrors), N_c -, N'_c - and N''_c -color D4-branes (and their mirrors) as well as $O6^+$ -plane(0123789), $O6^-$ -plane(0123789), eight half D6-branes(0123789) and the middle NS5'-brane ⁶. See also the relevant works [24, 25, 26] which have dealt with the gauge theory realized by the first factor in (3.1) with an antisymmetric tensor, a conjugate symmetric tensor, fundamentals and antifundamentals.

Let us place an $O6^+$ -plane at the origin $x^6 = 0$ and let us denote the x^6 coordinates for the $NS5_L$ -brane, the NS5'-brane and the $NS5_R$ -brane by $x^6 = y_1, y_1 + y_2, y_1 + y_2 + y_3$ respectively. Their mirrors can be understood similarly. The N_c D4-branes are suspended between the $NS5_L$ -brane and its mirror, the N'_c D4-branes are suspended between the $NS5_L$ -brane and the NS5'-brane (and their mirrors), and the N''_c D4-branes are suspended between the NS5'-brane and the $NS5_R$ -brane (and their mirrors). The fields A and \tilde{S} correspond to 4-4 strings connecting the N_c -color D4-branes with $x^6 < 0$ with N_c -color D4-branes with $x^6 > 0$. We draw this brane configuration in Figure 7A ⁷ for the vanishing mass for the bifundamentals.

Let us discuss three different magnetic gauge theories by taking each mass deformation.

3.1 $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(N'_c) \times SU(N''_c)$ magnetic theory

Let us take the mass deformation by moving the $(N'_c - N''_c)$ D4-branes between the $NS5_L$ -brane and the NS5'-brane to $+v$ direction in the electric theory and consider the Seiberg dual for the first gauge group.

- Mass deformation by F and \tilde{F}

There is no electric superpotential corresponding to the Figure 7A. It is known in [20] that the classical superpotential without $NS5_R$ -brane (and its mirror) for the particular orientation for NS-branes in Figure 7A is vanishing. Now we are adding the $NS5_R$ -brane (and its mirrors)

⁶When we say about NS-branes (NS5-brane or NS5'-brane) in this section, those NS-branes are located at $x^6 > 0$. We assume that their mirrors behave according to \mathbf{Z}_2 symmetry of $O6^+$ -plane. That is, there exist three NS-branes: $NS5_L$ -brane, NS5'-brane and $NS5_R$ -brane from Figure 6A.

⁷According to the observation of [24, 25, 26], this “fork” brane configuration contains the NS5'-brane embedded in an $O6$ -plane at $x^7 = 0$. This NS5'-brane divides the $O6$ -plane into two separated regions corresponding to positive x^7 and negative x^7 . Then RR charge of the $O6$ -plane jumps from -4 to $+4$. Furthermore, eight semi-infinite D6-branes are present in the positive x^7 region. This is necessary for the vanishing of the six dimensional anomaly. Further discussions on the gauge symmetry or flavor symmetry of this brane configuration can be found in [3] or [15].

into the brane configuration of [20] together with N_c'' D4-branes. Since the angle between the NS5'-brane and the extra $NS5_R$ -brane is $\frac{\pi}{2}$ in Figure 7A, the mass for the extra adjoint field μ'' corresponding to the gauge group $SU(N_c'')$ becomes large and after integrating out this adjoint field, then the extra classical superpotential term which has a factor $\frac{1}{\mu''}$ will vanish.

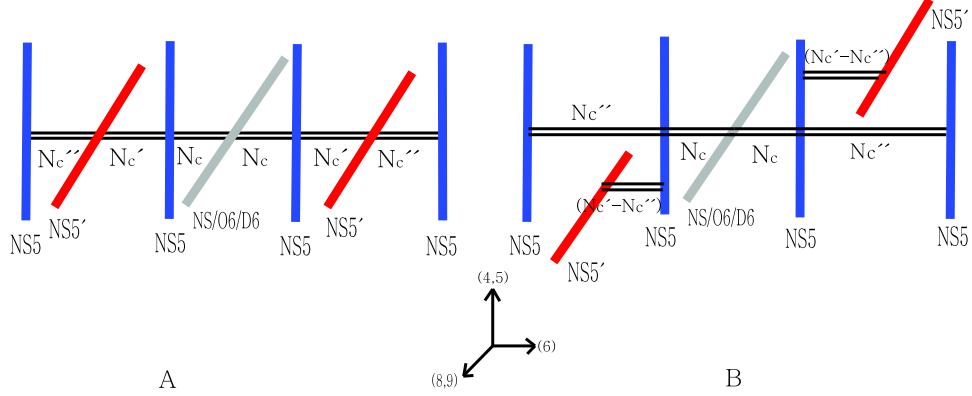


Figure 7: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ and an antisymmetric tensor A , a conjugate symmetric tensor \tilde{S} , bifundamentals $F, \tilde{F}, G, \tilde{G}$ and fundamentals \hat{Q} with vanishing(7A) and nonvanishing(7B) mass for the bifundamentals F and \tilde{F} . The NS5'-brane with $(N'_c - N''_c)$ D4-branes is moving to $+v$ direction in 7B(and their mirrors to $-v$ direction) and the remaining N''_c D4-branes are recombined with those D4-branes connecting between NS5'-brane and $NS5_R$ -brane in 7B. The notation for NS/O6/D6 refers to a combination of a middle NS5'-brane, $O6^+$ -plane with $x^7 \leq 0$, $O6^-$ -plane with $x^7 \geq 0$ and eight half D6-branes with $x^7 \geq 0$.

Now let us deform this theory. Displacing the two NS5'-branes relative each other in the $+v$ direction corresponds to turning on a quadratic superpotential for the bifundamentals F and \tilde{F} where the Φ' is a meson field

$$W_{def} = m F \tilde{F} (\equiv m \text{tr } \Phi'), \quad m = \frac{\Delta x}{\ell_s^2}$$

where the first gauge group indices in F and \tilde{F} are contracted and the mass m is related to the relative displacement Δx between the middle NS5'-brane and the NS5'-brane. The gauge-singlet Φ' , which has two free indices on the gauge group $SU(N'_c)$, for the gauge group is in the adjoint representation for the second gauge group $SU(N'_c)$, i.e., $(\mathbf{1}, (\mathbf{N}'_c - \mathbf{N}''_c)^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the gauge group where the gauge group $SU(N'_c)$ is broken to $SU(N'_c - N''_c)$. That is, the gauge singlet Φ' is a $(N'_c - N''_c) \times (N'_c - N''_c)$ matrix implying that only $(N'_c - N''_c)$ D4-branes participating in the mass deformation. The NS5'-brane together with $(N'_c - N''_c)$ -color D4-branes is moving to the $+v$ direction for fixed other branes during this particular mass

deformation (and their mirrors to $-v$ direction). We assume that the number of colors satisfies

$$2N'_c \geq N_c - 4 \geq N'_c + N''_c.$$

We draw this $\mathcal{N} = 1$ supersymmetric brane configuration in Figure 7B for nonvanishing mass for the bifundamentals by moving the NS5'-brane with $(N'_c - N''_c)$ color D4-branes to the $+v$ direction and their mirrors to $-v$ direction. The geometric configuration for three NS-branes in Figure 7B is exactly the same as the first three NS-branes in Figure 1B of [17].

- Seiberg dual

Let us apply the Seiberg dual to the first gauge group $SU(N_c)$ factor. Starting from Figure 7B and moving the $NS5_L$ -brane to the left all the way past the middle NS5'-brane (and the mirror of $NS5_L$ -brane to the right of the middle NS5'-brane), one obtains the Figure 8A. Before arriving at the Figure 8A, there exists an intermediate step where the \tilde{N}_c D4-branes are connecting between the $NS5_L$ -brane and its mirror, $(N'_c - N''_c)$ D4-branes are connecting between the $NS5_L$ -brane and NS5'-brane (and their mirrors) as well as N''_c D4-branes between NS5'-brane and the $NS5_R$ -brane. By introducing $(N'_c - N''_c)$ D4-branes and $(N'_c - N''_c)$ anti-D4-branes between $NS5_L$ -brane and its mirror, one gets the final Figure 8A where we are left with $(N'_c - N''_c - \tilde{N}_c)$ anti-D4-branes between the $NS5_L$ -brane and its mirror.

When two NS5'-branes in Figure 8A are close to each other, it becomes the meta-stable brane configuration Figure 8B by realizing that the number of $(N'_c - N''_c)$ D4-branes connecting between $NS5_L$ -brane and NS5'-brane can be rewritten as $(N_c - N'_c - N''_c - 4)$ plus \tilde{N}_c .

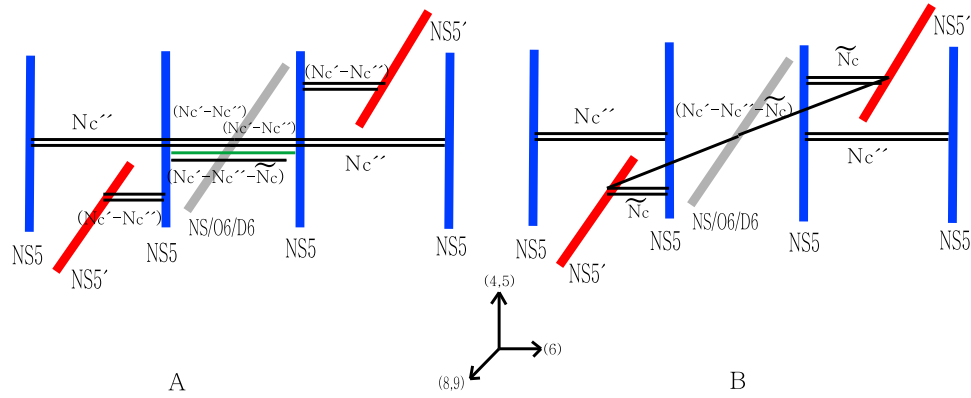


Figure 8: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(\tilde{N}_c = 2N'_c - N_c + 4) \times SU(N'_c) \times SU(N''_c)$ corresponding to Figure 7B with D4- and $\overline{D4}$ -branes (8A) and with a misalignment between D4-branes (8B) when the NS5'-branes are close to each other. The number of tilted D4-branes in 8B can be written as $N_c - N'_c - N''_c - 4 = (N'_c - N''_c) - \tilde{N}_c$.

The gauge group is given by

$$SU(\tilde{N}_c = 2N'_c - N_c + 4) \times SU(N'_c) \times SU(N''_c) \quad (3.2)$$

where the number of dual color ⁸ can be obtained from the linking number counting, as done in [20, 15]. Note that the number of flavors of D6-branes in [20] is zero because in the present case, there are no D6-branes. The matter contents are the flavor singlet f in the bifundamental representation $(\tilde{N}_c, \overline{N'_c}, \mathbf{1})$ and its complex conjugate field \tilde{f} in the bifundamental representation $(\tilde{N}_c, N'_c, \mathbf{1})$, and the gauge singlet $\Phi' \equiv F\tilde{F}$ in the representation for $(\mathbf{1}, (N'_c - N''_c)^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$, under the dual gauge group where the gauge group $SU(N'_c)$ is broken to $SU(N'_c - N''_c)$. There are also the antisymmetric flavor a , the conjugate symmetric flavor \tilde{s} and eight fundamentals \hat{q} for $SU(\tilde{N}_c)$ as well as G and \tilde{G} .

- Meta-stable brane configuration

Then the dual magnetic superpotential, by adding the mass term for the bifundamental, is given by

$$W_{dual} = \Phi' f \tilde{f} + m \text{tr} \Phi' + \hat{q} \tilde{s} \hat{q}. \quad (3.3)$$

Here the magnetic fields f and \tilde{f} correspond to 4-4 strings connecting the \tilde{N}_c -color D4-branes (that are connecting between the $NS5_L$ -brane and the $NS5'$ -brane in Figure 8B) with N'_c -flavor D4-branes (which are realized as corresponding D4-branes in Figure 8A). Among these N'_c -flavor D4-branes, only the strings ending on the upper \tilde{N}_c D4-branes and on the tilted $(N'_c - N''_c - \tilde{N}_c)$ D4-branes in Figure 8B enter the above cubic superpotential term. Note that the summation of these D4-branes is equal to $(N'_c - N''_c)$. Although the $(N'_c - N''_c)$ D4-branes in Figure 8A cannot move any directions, the tilted $(N'_c - N''_c - \tilde{N}_c)$ -flavor D4-branes can move w direction in Figure 8B (and its mirrors). The remaining upper \tilde{N}_c D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

$$N'_c - N''_c = (N_c - N'_c - N''_c - 4) + \tilde{N}_c.$$

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 8A by moving the upper $NS5'$ -brane together with $(N'_c - N''_c)$ color D4-branes into the origin $v = 0$ (and their mirrors). Then the number of dual colors for D4-branes becomes \tilde{N}_c between the $NS5_L$ -brane and its mirror, N'_c between $NS5_L$ -brane and the $NS5'$ -brane and N''_c between $NS5'$ -brane and $NS5_R$ -brane.

The brane configuration in Figure 8A is stable as long as the distance Δx between the upper $NS5'_L$ -brane and the middle $NS5'$ -brane is large. If they are close to each other then this brane configuration is unstable to decay and it becomes the meta-stable brane configuration

⁸This duality symmetry in field theory side was tested in [11] also and the brane configuration in type IIA string theory for the supersymmetric magnetic theory was studied in [24, 25, 26]. Moreover, the nonsupersymmetric meta-stable brane configuration was found in [15] where the correct movement of the branes were crucial.

in Figure 8B. Since the two NS5'-branes are located at different sides of NS5-brane in Figure 8B, contrary to the previous cases, for the DBI computation, this fact should be taken into account. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (3.2) and superpotential (3.3). When the two NS5'-branes which are connected by \tilde{N}_c D4-branes are replaced by two coincident $(N'_c - N''_c)$ D6-branes, the brane configuration of Figure 8B (with the rotation of $NS5_R$ -brane by $\frac{\pi}{2}$) is the same as the Figure 7 (with opposite movement of D6-branes to v direction) studied in [20] where the gauge group is given by $SU(2n_f + 2n'_c - n_c + 4) \times SU(n'_c)$ with n_f fundamentals, bifundamentals, an antisymmetric flavor, a conjugate symmetric flavor, and various gauge singlets. Then the our N_c corresponds to the n_c , the number $(N'_c - N''_c)$ corresponds to n_f , and our N''_c corresponds to the n'_c of [20].

The N''_c D4-branes between the $NS5_L$ -brane and the $NS5_R$ -brane can slide along the v direction (and their mirrors to the opposite direction).

- Gauge theory at small Δx

The gauge couplings for the gauge group factors are given by similarly and the superpotential corresponding to Figures 8A and 8B is given by

$$W_{dual} = h\Phi' f \tilde{f} - h\mu^2 \text{tr} \Phi' + \hat{q} \tilde{s} \hat{q}, \quad h^2 = g_{2,mag}^2$$

and the mass parameter μ^2 is the same as before. See also the relevant papers [15, 9]. Then the product $f\tilde{f}$ is a $\tilde{N}_c \times \tilde{N}_c$ matrix where the second gauge group indices for f and \tilde{f} are contracted with those of Φ' while Φ' is a $(N'_c - N''_c) \times (N'_c - N''_c)$ matrix. Although the field f itself is an antifundamental in the second gauge group, the product $f\tilde{f}$ has the same representation with the product of dual quarks and the second gauge group indices for the field Φ' play the role of the flavor indices.

Therefore, the F-term equation, the derivative of W_{dual} with respect to the meson field Φ' cannot be satisfied if the $(N'_c - N''_c)$ exceeds \tilde{N}_c . So the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. That is, there are five equations from F-term conditions: $f^a \tilde{f}_b - \mu^2 \delta_b^a = 0$, $\Phi' f = 0$, $\tilde{f} \Phi' = 0$, $\hat{q} \tilde{s} = 0$, and $\hat{q} \hat{q} = 0$. Then the solutions for these are given by

$$\begin{aligned} \langle f \rangle &= \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}_c} \\ 0 \end{pmatrix}, \quad \langle \tilde{f} \rangle = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}_c} & 0 \end{pmatrix}, \quad \langle \Phi' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi'_0 \mathbf{1}_{(N'_c - N''_c - \tilde{N}_c)} \end{pmatrix}, \\ \langle \hat{q} \rangle &= 0, \quad \langle \tilde{s} \rangle = 0. \end{aligned}$$

One can expand around the solutions, as done in [9]. Although there exists an extra last term in (3.3), this does not contribute to the one loop result. At one loop, the effective potential

$V_{eff}^{(1)}$ for Φ'_0 leads to the positive value for $m_{\Phi'_0}^2$ implying that these vacua are stable. The gauge theory analysis is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C .

3.2 $\mathcal{N} = 1$ $SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$ magnetic theory

Let us consider the Seiberg dual for the second gauge group and take the mass deformation by moving the N''_c D4-branes between the the NS5-brane and $NS5'_R$ -brane to $+v$ direction. See also the Figure 9. Since the discussion is analogous to previous subsection 2.1, we mention the main results briefly.

- Seiberg dual and mass deformation

By applying the Seiberg dual to the second gauge group $SU(N'_c)$ factor in (3.1) and interchanging the $NS5_L$ -brane and the NS5'-brane(and their mirrors) from Figure 7A, one obtains the Figure 9A.

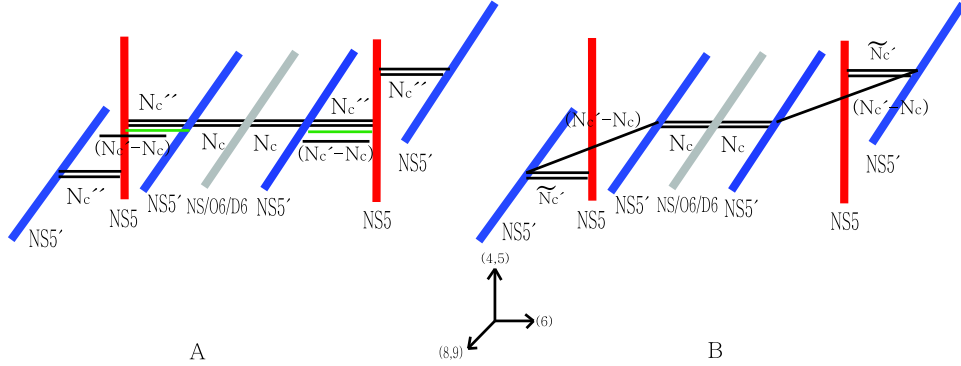


Figure 9: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(N_c) \times SU(\tilde{N}'_c = N_c + N''_c - N'_c) \times SU(N''_c)$ with D4- and $\overline{D4}$ -branes(9A) and with a misalignment between D4-branes(9B) when the NS5'-branes are close to each other. The number of tilted D4-branes is equal to $N'_c - N_c = N''_c - \tilde{N}'_c$ in 9B.

When two NS5'-branes in Figure 9A are close to each other, it becomes the meta-stable brane configuration Figure 9B by realizing that the number of N''_c D4-branes connecting between NS5-brane and $NS5'_R$ -brane can be rewritten as $(N'_c - N_c)$ plus \tilde{N}'_c . Now we draw this $\mathcal{N} = 1$ supersymmetric magnetic brane configuration in Figure 9. We assume that the number of colors satisfies (2.3). The brane configuration consisting of NS5-brane and two NS5'-branes in Figure 9 is exactly the same as those in Figure 15 of [17] if we replace the NS5/O6/D6-branes with $O6^+$ -plane. There is an other mass deformation by moving the middle NS5'-brane located at $x^6 = 0$ to $+v$ direction.

The dual gauge group is given by (2.4) and for convenience we rewrite it here

$$SU(N_c) \times SU(\tilde{N}'_c = N_c + N''_c - N'_c) \times SU(N''_c). \quad (3.4)$$

The matter contents are the field f charged under $(\mathbf{N}_c, \overline{\mathbf{N}'_c}, \mathbf{1})$, a field g charged under $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}''_c})$, and their conjugates \tilde{f} and \tilde{g} under the dual gauge group (3.4) and the gauge-singlet Φ'' for the second dual gauge group in the adjoint representation for the third dual gauge group, i.e., $(\mathbf{1}, \mathbf{1}, \mathbf{N}''_c^2 - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the dual gauge group. Then the Φ'' is a $N''_c \times N''_c$ matrix. There are also the antisymmetric flavor A , the conjugate symmetric flavor \tilde{S} and eight fundamentals \hat{Q} for $SU(N_c)$.

- Meta-stable brane configuration

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 9A by moving the upper NS5'-brane together with N''_c color D4-branes into the origin $v = 0$ (and their mirrors). Then the number of dual colors for D4-branes becomes N_c between the $NS5'_L$ -brane and its mirror, \tilde{N}'_c between $NS5'_L$ -brane and NS5-brane and N''_c between NS5-brane and $NS5'_R$ -brane.

The brane configuration in Figure 9A is stable as long as the distance Δx between the upper NS5'-brane and the lower NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 9B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group and superpotential. The upper tilted $(N''_c - \tilde{N}'_c)$ flavor D4-branes of straight brane configuration of Figure 9B bend since there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action. As mentioned in [9], the two NS5'-branes are located at different side of NS5-brane in Figure 9B and the DBI action computation for this bending curve should be taken into account.

The N_c D4-branes between the $NS5'_L$ -brane and its mirror can slide along the w direction. When y_1 goes to zero, this meta-stable brane configuration of Figure 9B where there are three NS5'-branes at the origin reduces to the Figure 6 of [9] where there is a single NS5'-brane at the origin.

- Gauge theory at small Δx

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (3.4) and the gauge couplings for the three gauge group factors are given by the expressions in subsection 2.1.

The dual gauge theory has a meson field Φ'' and bifundamentals f, \tilde{f}, g and \tilde{g} under the dual gauge group (3.4) and the superpotential corresponding to Figures 9A and 9B is given by the expressions in subsection 2.1. Then $g\tilde{g}$ is a $\tilde{N}'_c \times \tilde{N}'_c$ matrix where the third gauge

group indices for g and \tilde{g} are contracted with those of Φ'' while Φ'' is a $N_c'' \times N_c''$ matrix. The product $g\tilde{g}$ has the same representation for the product of quarks and moreover, the third gauge group indices for the field Φ'' play the role of the flavor indices.

When the upper NS5'-brane(or $NS5'_R$ -brane) is replaced by coincident N_c'' D6-branes in Figure 9B, this brane configuration looks similar to the Figure 7 found in [20] where the gauge group was given by $SU(n_c) \times SU(n'_f + n_c - n'_c)$ with n'_f fundamentals, bifundamentals, an antisymmetric flavor, a conjugate symmetric flavor, and gauge singlets. Then the present N_c corresponds to the n_c , N'_c corresponds to n'_c , and N_c'' corresponds to the n'_f of [20]. The location of $NS5'_R$ -brane is in the right hand side of NS5-brane while the location of n'_f D6-branes of Figure 7 in [20] is in the left hand side of NS5-brane.

3.3 $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'')$ magnetic theory

Let us consider the Seiberg dual for the third gauge group and take the mass deformation by moving the $(N'_c - N_c)$ D4-branes between the $NS5'_L$ -brane and the NS5-brane to $+v$ direction. See also the Figure 10. Since the discussion is analogous to previous subsection 2.2, we mention the main results briefly.

- Seiberg dual and mass deformation by G and \tilde{G}

By applying the Seiberg dual to the third gauge group $SU(N_c'')$ factor in (3.1) and interchanging the NS5'-brane and the $NS5_R$ -brane(and their mirrors), one obtains the Figure 10A.

When two NS5'-branes in Figure 10A are close to each other, then it leads to Figure 10B by realizing that the number of $(N'_c - N_c)$ D4-branes connecting between $NS5'_L$ -brane and NS5-brane can be rewritten as $(N_c'' - N_c)$ plus \tilde{N}_c'' . The brane configuration consisting of NS5-brane and two NS5'-branes in Figure 10B is exactly the same as the last three NS-branes in Figure 5B'' of [17]. The brane configuration in Figure 10 is exactly the same as those in Figure 16 of [17] if we replace the NS5/O6/D6 with $O6^+$ -plane.

The dual gauge group is given by (2.9) and we repeat here

$$SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'' = N'_c - N_c''). \quad (3.5)$$

The matter contents are the field F charged under $(\mathbf{N}_c, \overline{\mathbf{N}}_c', \mathbf{1})$, a field g charged under $(\mathbf{1}, \mathbf{N}_c', \overline{\mathbf{N}}_c'')$ and their conjugates \tilde{F} and \tilde{g} under the dual gauge group (3.5) and the gauge-singlet Φ' which is in the adjoint representation for the second dual gauge group, in other words, $(\mathbf{1}, (\mathbf{N}_c' - \mathbf{N}_c)^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the dual gauge group (3.5) where the gauge group $SU(N'_c)$ is broken to $SU(N'_c - N_c)$. Then the Φ' is a $(N'_c - N_c) \times (N'_c - N_c)$ matrix. Only $(N'_c - N_c)$ D4-branes are participating in the mass deformation. There are also

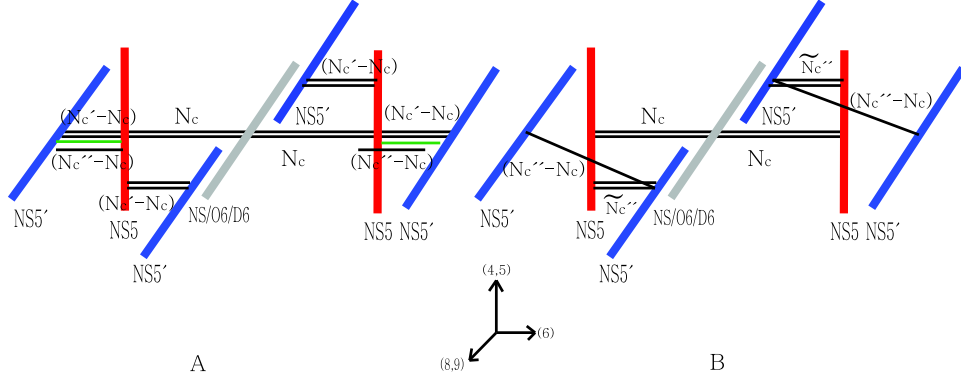


Figure 10: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'' = N'_c - N_c'')$ with D4- and $\overline{D4}$ -branes(10A) and with a misalignment between D4-branes(10B) when the NS5'-branes are close to each other. The number of tilted D4-branes is equal to $N_c'' - N_c = (N'_c - N_c) - \tilde{N}_c''$ in 10B.

the antisymmetric flavor A , the conjugate symmetric flavor \tilde{S} and eight fundamentals \hat{Q} for $SU(N_c)$.

- Meta-stable brane configuration

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 10A by moving the upper NS5'-brane together with $(N'_c - N_c)$ color D4-branes into the origin $v = 0$ (and their mirrors). Then the number of dual colors for D4-branes becomes N_c between the $NS5'_L$ -brane and its mirror, N'_c between the $NS5'_L$ -brane and the NS5-brane and \tilde{N}_c'' between NS5-brane and $NS5'_R$ -brane.

When the upper NS5'-brane (or $NS5'_L$ -brane) is replaced by coincident $(N'_c - N_c)$ D6-branes in Figure 10B, this brane configuration looks similar to the Figure 5 found in [20] where the gauge group was given by $SU(n_c) \times SU(n'_f + n_c - n'_c)$ with n'_f fundamentals, bifundamentals, a symmetric flavor, a conjugate symmetric flavor, and gauge singlets. The only difference between these two brane configurations appears at the origin $x^6 = 0$. Then the present N_c corresponds to the n_c , $(N'_c - N_c)$ corresponds to n'_f , and N_c'' corresponds to the n'_c of [20].

- Gauge theory at small Δx

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (3.5) and the gauge couplings for the three gauge group factors are given by the expressions in subsection 2.2. The dual gauge theory has a meson field Φ' and bifundamentals f, \tilde{f}, g and \tilde{g} under the dual gauge group (3.5) and the superpotential corresponding to Figures 10A and 10B is given by the one in subsection 2.2. Then $g\tilde{g}$ is a $\tilde{N}_c'' \times \tilde{N}_c''$ matrix where the second gauge group indices for g and \tilde{g} are contracted with those of Φ' while Φ' is a $(N'_c - N_c) \times (N'_c - N_c)$ matrix. The product $g\tilde{g}$

has the same representation for the product of quarks and moreover, the second gauge group indices for the field Φ' play the role of the flavor indices.

4 Conclusions and outlook

The meta-stable brane configurations we have found are summarized by Figures 3, 5, 6, 8, 9, 10. If we replace the upper $NS5'$ -brane in Figures 6B(with a rotation of $NS5'_R$ -brane) and 8B(with a rotation of $NS5_R$ -brane) with the coincident D6-branes, those brane configurations become nonsupersymmetric minimal energy brane configurations in the Figures 4 and 6 found in [20] previously. By changing some of the NS-branes, D-branes or O-planes from the remaining Figures 3, 5, 9 and 10, one obtains the brane configurations Figures 12, 14, 15 and 16 given by [17] respectively.

Some different directions on the meta-stable vacua are present in recent relevant works [27]-[53] where some of them are described in the type IIB string theory. It would be very interesting to find out how the meta-stable brane configurations from type IIA string theory including the present work are related to those brane configurations from type IIB string theory.

Acknowledgments

I would like to thank KIAS(Korea Institute for Advanced Study) and Harvard High Energy Theory Group for hospitality where this work was undertaken. This work was supported by grant No. R01-2006-000-10965-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

References

- [1] K. Intriligator, N. Seiberg and D. Shih, “Dynamical SUSY breaking in meta-stable vacua,” JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].
- [2] K. Intriligator and N. Seiberg, “Lectures on Supersymmetry Breaking,” [arXiv:hep-ph/0702069].
- [3] A. Giveon and D. Kutasov, “Brane dynamics and gauge theory,” Rev. Mod. Phys. **71**, 983 (1999) [arXiv:hep-th/9802067].
- [4] H. Ooguri and Y. Ookouchi, “Meta-stable supersymmetry breaking vacua on intersecting branes,” Phys. Lett. B **641**, 323 (2006) [arXiv:hep-th/0607183].

- [5] S. Franco, I. Garcia-Etxebarria and A. M. Uranga, “Non-supersymmetric meta-stable vacua from brane configurations,” JHEP **0701**, 085 (2007) [arXiv:hep-th/0607218].
- [6] I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and D. Shih, “A note on (meta)stable brane configurations in MQCD,” JHEP **0611**, 088 (2006) [arXiv:hep-th/0608157].
- [7] C. Ahn, “Brane configurations for nonsupersymmetric meta-stable vacua in SQCD with adjoint matter,” Class. Quant. Grav. **24**, 1359 (2007) [arXiv:hep-th/0608160].
- [8] A. Giveon and D. Kutasov, “Gauge symmetry and supersymmetry breaking from intersecting branes,” [arXiv:hep-th/0703135].
- [9] C. Ahn, “Meta-Stable Brane Configurations by Adding an Orientifold-Plane to Giveon-Kutasov,” arXiv:0706.0042 [hep-th], to appear in JHEP.
- [10] C. Ahn, “M-theory lift of meta-stable brane configuration in symplectic and orthogonal gauge groups,” Phys. Lett. B **647**, 493 (2007) [arXiv:hep-th/0610025].
- [11] K. A. Intriligator, R. G. Leigh and M. J. Strassler, “New examples of duality in chiral and nonchiral supersymmetric gauge theories,” Nucl. Phys. B **456**, 567 (1995) [arXiv:hep-th/9506148].
- [12] E. Barnes, K. Intriligator, B. Wecht and J. Wright, “ $N = 1$ RG flows, product groups, and a-maximization,” Nucl. Phys. B **716**, 33 (2005) [arXiv:hep-th/0502049].
- [13] J. H. Brodie and A. Hanany, “Type IIA superstrings, chiral symmetry, and $N = 1$ 4D gauge theory dualities,” Nucl. Phys. B **506**, 157 (1997) [arXiv:hep-th/9704043].
- [14] C. Ahn, “Meta-stable brane configuration with orientifold 6 plane,” JHEP **0705**, 053 (2007) [arXiv:hep-th/0701145].
- [15] C. Ahn, “More on Meta-Stable Brane Configuration,” Class. Quant. Grav. **24**, 3603 (2007) [arXiv:hep-th/0702038].
- [16] C. Ahn and R. Tatar, “Geometry, D-branes and $N = 1$ duality in four dimensions with product gauge groups,” Phys. Lett. B **413**, 293 (1997) [arXiv:hep-th/9705106].
- [17] C. Ahn, “More Meta-Stable Brane Configurations without D6-Branes,” arXiv:0707.0092 [hep-th].
- [18] C. Ahn, “Meta-Stable Brane Configuration of Product Gauge Groups,” arXiv:0704.0121 [hep-th].

- [19] C. Ahn, “Meta-Stable Brane Configuration and Gauged Flavor Symmetry,” [arXiv:hep-th/0703015], to appear in MPLA.
- [20] C. Ahn, “Meta-stable brane configurations with five NS5-branes,” arXiv:0705.0056 [hep-th].
- [21] K. Landsteiner, E. Lopez and D. A. Lowe, “Supersymmetric gauge theories from branes and orientifold six-planes,” JHEP **9807**, 011 (1998) [arXiv:hep-th/9805158].
- [22] K. Landsteiner and E. Lopez, “New curves from branes,” Nucl. Phys. B **516**, 273 (1998) [arXiv:hep-th/9708118].
- [23] D. Shih, “Spontaneous R-Symmetry Breaking in O’Raifeartaigh Models,” [arXiv:hep-th/0703196].
- [24] K. Landsteiner, E. Lopez and D. A. Lowe, “Duality of chiral $N = 1$ supersymmetric gauge theories via branes,” JHEP **9802**, 007 (1998) [arXiv:hep-th/9801002].
- [25] I. Brunner, A. Hanany, A. Karch and D. Lust, “Brane dynamics and chiral non-chiral transitions,” Nucl. Phys. B **528**, 197 (1998) [arXiv:hep-th/9801017].
- [26] S. Elitzur, A. Giveon, D. Kutasov and D. Tsabar, “Branes, orientifolds and chiral gauge theories,” Nucl. Phys. B **524**, 251 (1998) [arXiv:hep-th/9801020].
- [27] J. J. Heckman and C. Vafa, “Geometrically Induced Phase Transitions at Large N ,” arXiv:0707.4011 [hep-th].
- [28] O. Aharony and S. Kachru, “Stringy Instantons and Cascading Quivers,” arXiv:0707.3126 [hep-th].
- [29] R. Tatar and B. Wetenhall, “Metastable Vacua and Complex Deformations,” arXiv:0707.2712 [hep-th].
- [30] A. Amariti, L. Girardello and A. Mariotti, “Meta-stable A_n quiver gauge theories,” arXiv:0706.3151 [hep-th].
- [31] D. Malyshev, “Del Pezzo singularities and SUSY breaking,” arXiv:0705.3281 [hep-th].
- [32] L. Ferretti, “R-symmetry breaking, runaway directions and global symmetries in O’Raifeartaigh models,” arXiv:0705.1959 [hep-th].

- [33] J. Marsano, K. Papadodimas and M. Shigemori, “Nonsupersymmetric brane / antibrane configurations in type IIA and M theory,” arXiv:0705.0983 [hep-th].
- [34] H. Ooguri, Y. Ookouchi and C. S. Park, “Metastable Vacua in Perturbed Seiberg-Witten Theories,” arXiv:0704.3613 [hep-th].
- [35] T. Kawano, H. Ooguri and Y. Ookouchi, “Gauge Mediation in String Theory,” arXiv:0704.1085 [hep-th].
- [36] I. Garcia-Etxebarria, F. Saad and A. M. Uranga, “Supersymmetry breaking metastable vacua in runaway quiver gauge theories,” arXiv:0704.0166 [hep-th].
- [37] K. Intriligator, N. Seiberg and D. Shih, “Supersymmetry Breaking, R-Symmetry Breaking and Metastable Vacua,” [arXiv:hep-th/0703281].
- [38] S. Hirano, “Models with Quartic Potential of Dynamical SUSY Breaking in Meta-Stable Vacua,” JHEP **0705**, 064 (2007) [arXiv:hep-th/0703272].
- [39] S. Murthy, “On supersymmetry breaking in string theory from gauge theory in a throat,” [arXiv:hep-th/0703237].
- [40] R. Argurio, M. Bertolini, S. Franco and S. Kachru, “Metastable vacua and D-branes at the conifold,” [arXiv:hep-th/0703236].
- [41] Y. E. Antebi and T. Volansky, “Dynamical supersymmetry breaking from simple quivers,” [arXiv:hep-th/0703112].
- [42] M. Wijnholt, “Geometry of particle physics,” [arXiv:hep-th/0703047].
- [43] J. J. Heckman, J. Seo and C. Vafa, “Phase structure of a brane/anti-brane system at large N,” [arXiv:hep-th/0702077].
- [44] A. Amariti, L. Girardello and A. Mariotti, “On meta-stable SQCD with adjoint matter and gauge mediation,” [arXiv:hep-th/0701121].
- [45] R. Kitano, H. Ooguri and Y. Ookouchi, “Direct mediation of meta-stable supersymmetry breaking,” Phys. Rev. D **75**, 045022 (2007) [arXiv:hep-ph/0612139].
- [46] R. Tatar and B. Wetenhall, “Metastable vacua, geometrical engineering and MQCD transitions,” JHEP **0702**, 020 (2007) [arXiv:hep-th/0611303].

- [47] H. Verlinde, “On metastable branes and a new type of magnetic monopole,” [arXiv:hep-th/0611069].
- [48] M. Aganagic, C. Beem, J. Seo and C. Vafa, “Geometrically induced metastability and holography,” [arXiv:hep-th/0610249].
- [49] R. Argurio, M. Bertolini, S. Franco and S. Kachru, “Gauge/gravity duality and meta-stable dynamical supersymmetry breaking,” JHEP **0701**, 083 (2007) [arXiv:hep-th/0610212].
- [50] A. Amariti, L. Girardello and A. Mariotti, “Non-supersymmetric meta-stable vacua in SU(N) SQCD with adjoint matter,” JHEP **0612**, 058 (2006) [arXiv:hep-th/0608063].
- [51] S. Franco, I. Garcia-Etxebarria and A. M. Uranga, “Non-supersymmetric meta-stable vacua from brane configurations,” JHEP **0701**, 085 (2007) [arXiv:hep-th/0607218].
- [52] H. Ooguri and Y. Ookouchi, “Landscape of supersymmetry breaking vacua in geometrically realized gauge theories,” Nucl. Phys. B **755**, 239 (2006) [arXiv:hep-th/0606061].
- [53] S. Franco and A. M. Uranga, “Dynamical SUSY breaking at meta-stable minima from D-branes at obstructed geometries,” JHEP **0606**, 031 (2006) [arXiv:hep-th/0604136].